The effect of continuum scattering processes on spectral line formation

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Abstract

The effect of scattering processes in the continuum on the formation of spectral lines in a static atmosphere with an arbitrary distribution of the internal energy sources is investigated using Ambartsumian's principle of invariance. Spectral line profiles are calculated to illustrate the effect the assumption of the complete redistribution on atoms and coherent scattering in continuum may have on the emergent intensity. The one-dimensional case is considered for simplicity.

1 Introduction

In the study of the formation of spectral lines, an interesting problem is the transfer of radiation in a medium consisting of centers at which scattering occurs for all frequencies. This class of problems includes in particular that of the formation of spectral lines in one-dimensional, semi-infinite media. In his pionering paper Schuster[1] showed that under some circumstances scattering of photons in continuum can lead to the formation of emission lines in the spectrum. The formation mechanism of emission lines due to scattering in continuum is simple. A strong scattering source in the continuum may force the photons to be more scattered than absorbed in the certain part of the spectrum and will therefore lead to a decrease in the source function in the continuum. Suppose that the radiation field in a spectral line is in local thermodynamical equilibrium (LTE) with the local medium so that the line photons have much more chance of being absorbed and destroyed in the medium than photons in the continuum. The increase in the efficiency of scattering causes the radiation field in the continuum to decrease in the meantime, thereby keeping the flux in the spectral line unaffected because of the thermal radiation of the medium. Thus the spectral line will appear in emission.

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2 Spectral lines formed in an isothermal medium

Recently Israelian and Nikoghossian[2] investigated the diffusion of isotropic and coherent radiation in a semi-infinite medium by the probabilistic approach. The probabilistic method is based on the concept of a probability of quantum exit from a medium and has been successfully applied to treat many classical problems in a theory of radiative transfer[3]. The principle of optical reversibility, together with the principle of invariance, allows very complex problems to be handled when the probabilistic approach is employed.

Let us consider a one-dimensional, semi-infinite, isothermal medium with a distribution of energy sources:

$$\epsilon(\tau, x) = u(x)B[T(\tau)], \tag{1}$$

where $u(x) = (1-\lambda)\alpha(x) + \beta$. Here we denote x as a non-dimensional frequency ($x = \delta\nu/\delta\nu_D$, where $\delta\nu_D$ is a Doppler width), $\alpha(x)$ as the normalized absorption coefficient profile in the line, $B[T(\tau)]$ is the Planck function, which depends on the optical depth, τ, relative to the central wavelength of the line through the temperature, T, λ is the probability of photon re-emission, β is the ratio of the absorption coefficient in the continuum to that at the center of the spectral line. We have also introduced[2] a parameter γ, the ratio of the scattering coefficient in the continuum to the absorption coefficient at the line center. Let us assume that the medium has an infinitely large optical thickness so that the addition of a layer of small optical thickness will not change the reflection coefficient of the medium. This *principle of invariance* of Ambartsumian[4,5] allows one to obtain the following equation:

$$\frac{2}{\lambda}[v(x') + v(x)]\rho(x, x') = r(x, x') + \int r(x', x'')\rho(x'', x)dx''$$
$$+ \int r(x'', x)\rho(x', x'')dx'' + \int \rho(x', x'')dx'' \int r(x'', x''')\rho(x''', x)dx''', \tag{2}$$

where $v(x) = \alpha(x) + \beta + \gamma$, $\rho(x, x')$ and $r(x, x')$ are the reflection and redistribution functions, respectively. The reflection function, $\rho(x, x')$, has the following probabilistic meaning: if a photon of frequency x' is incident on the medium, then $\rho(x, x')dx$ is the probability that, after multiple scattering, a photon is reflected from the medium with a frequency in the interval x,$x + dx$.

Hereafter, all integrations over x will be carried out from $-\infty$ to $+\infty$. In general, there can be several sources of absorption/scattering in the medium, in which case $r(x, x')$ must be considered as a sum of redistribution functions due to different centers (like atoms, electrons, dust particles, molecules, plasmons,

etc.). Let us also add pure scattering centers in the medium and denote their redistribution function as $r_e(x,x')$. If these centers are free electrons and the scattering is incoherent[5] then

$$r_e(x,x') = \delta\nu_D{}^{-1}\pi^{-0.5}(e^{-x^2} - 2x\pi\int\limits_x^\infty exp(-u^2)du). \tag{3}$$

In case of the coherent scattering we have:

$$r_e(x,x') = \frac{\gamma}{\lambda}\delta(x-y). \tag{4}$$

The total redistribution function in the case of the complete redistribution of photons on the atoms and the coherent scattering on the electrons will be

$$r(x,x') = \alpha(x)\alpha(x') + \frac{\gamma}{\lambda}\delta(x-y). \tag{5}$$

The problem of determining the radiation field in a medium with or without energy sources under broad assumptions concerning the elementary scattering process can be simplified if the redistribution function for atoms allows presentation in the form of a bilinear expansion[6,7,8]:

$$r(x,x') = \sum_{k=0}^{\infty} A_k\alpha_k(x)\alpha_k(x'), \tag{6}$$

where

$$A_k = \frac{1}{2k+1},\ \alpha_k(x) = \frac{1}{\pi^{0.25}2^k\sqrt{(2k)!}}e^{-x^2}H_{2k}(x)$$

and $H_{2k}(x)$ are the Hermit polynomials. Note that with only the first term taken in the expansion (6), we arrive at complete frequency redistribution, i.e., there is no correlation between the frequency of the incoming and the scattered quantum in the observer's frame of refernce. Such a representation can be obtained by expanding the function $r(x,x')$ with respect to its eigenfunctions over $(-\infty,\infty)$. In general, the function $r(x,x')$ can be replaced by an finite sum of its expansion with respect to its eigenfunctions over $(-\infty,\infty)$:

$$r(x,x') \approx r_N(x,x') = \sum_{k=0}^{N} \frac{\chi_k(x)\chi_k(x')}{\lambda_k}, \tag{7}$$

where the functions $\chi_k(x)$ are the normalized solutions of the equations

$$\chi_k(x) = \lambda_k \int_{-\infty}^{\infty} r(x, x')\chi_k(x')dx'. \tag{8}$$

For fixed N, this partial sum gives the best mean-square solution approximation in the (x', x)-plane to the function $r(x', x)$ among all possible representations. In the case of representation (7), the accuracy of the fulfillment of condition (8) agrees with the accuracy of the approximation of the function $r(x, x')$ by the partial sum $r_N(x, x')$. Different methods (like the method of least squares, the method of moments, etc.) exist for the approximate construction of the eigenfunctions of the kernel $r(x, x')$ and we shall not dwell on this point.

Suppose that the function $r_e(x, x')$ can be expanded as well:

$$r_e(x, x') = \sum_{m=0}^{\infty} B_m \omega_m(x)\omega_m(x'). \tag{9}$$

Then in a more general case we will have a sum of functions (6) and (9). If the sum of representations (6) and (7) for the redistribution function, $r(x, x')$, is used, then the integral equation (2) can be rewritten as:

$$\rho(x, x') = \frac{\lambda}{2}\left[\sum_{k=0}^{\infty} A_k \frac{\varphi_k(x)\varphi_k(x')}{v(x)+v(x')} + \sum_{i=0}^{\infty} B_i \frac{\psi_i(x)\psi_i(x')}{v(x)+v(x')}\right], \tag{10}$$

where the functions φ_k and $\psi_i(x)$ satisfy the set of functional equations providing a generalization of Ambartsumian's functional equation for non-coherent scattering:

$$\varphi_k(x) = \alpha_k(x) + \int_{-\infty}^{\infty} \alpha_k(z)\rho(x, z)dz, \tag{11}$$

$$\psi_i(x) = \omega_i(x) + \int_{-\infty}^{\infty} \omega_i(x)\rho(x, z)dz. \tag{12}$$

The solution of this system provides a reflection function, $\rho(x, x')$, which allows the evaluation of the profiles of spectral lines (since $R(x) = \int \rho(x, x')dx'$). Determination of the profiles of absorption lines in an isothermal atmosphere is equivalent to that for diffuse reflection of radiation from a semi-infinite

medium[3]. The reflection and absorption profiles in this case are related by

$$R(x) + \tilde{R}(x) = 1. \tag{13}$$

This relation enables one to reduce line-profile calculations for an isothermal atmosphere to the evaluation of the reflection function, $\rho(x, x')$.

In this article we present numerical results for the simplest case of complete redistribution of photons on atoms and a coherent scattering on free electrons. If last term in the equation (5) describes a coherent scattering in the continuum, then it can be written in the form

$$\frac{\gamma}{\lambda}\delta(x - x') = \frac{\gamma}{\lambda}\sum_{k=0}^{\infty}\alpha_k(x')\alpha_k(x), \tag{14}$$

using the set of the $\alpha_k(x)$ functions given in the expansion (6).

3 Spectral line formation in a non-isothermal medium

In the case of a non-isothermal medium we shall assume that the source function can be presented in the following form:

$$B[T(\tau)] = \sum_{n=0}^{\infty}\frac{B_n}{n!}(\beta\tau)^n, \tag{15}$$

where B_n is the n^{th} derivative of the Planck function. Let us denote by $Y(\tau, x', x)$ the probability that a photon with a frequency x' moving in the medium at an optical depth τ will leave it with a frequency in the range $(x, x + dx)$. Similarly, let $P(\tau, x', x)$ be the same probability but for photons absorbed at an optical depth τ. Application of the invariance principle and simple physical reasoning results in the following equations:

$$\frac{\partial Y(\tau, x', x)}{\partial \tau} = -v(x)Y(\tau, x', x) + \int_{-\infty}^{\infty} P(0, z, x)\,\alpha(z)\,Y(\tau, x', z)\,dz \tag{16}$$

$$\alpha(x)P(0, x', x) = \frac{\lambda}{2}r(x', x) + \frac{\lambda}{2}\int_{-\infty}^{\infty} r(x', z)\,\rho(z, x)\,dz \tag{17}$$

These equations can be easily derived when β=0 (Sobolev[4]) and $\beta \neq$0 (Nikoghossian and Haruthyunian[8]).

The intensity of the emergent radiation, $I(0,x)$, with the source function given by (15) can be presented in the form

$$I(0,x)=\sum_{n=0}^{\infty} B_n I_n(0,x), \tag{18}$$

where

$$I_n(0,x)=\int_{-\infty}^{\infty} u(x')dx' \int_{0}^{\infty} f_n(\tau)Y_(\tau,x',x)d\tau \tag{19}$$

and $f_n(\tau)=(\beta\tau)^n/n!$.

The spectral line contours for the mentioned distribution of the energy sources can be calculated with the formulae[7]

$$v(x)R_n(x)=\int_{-\infty}^{\infty} \alpha(x')P(0,x',x)R_n(x')dx'+\beta R_{n-1}(x). \tag{20}$$

Note that γ appears only on the left-hand side of this equation. The final profiles are

$$R(x)=\frac{\sum_n B_n R_n(x)}{\sum_n B_n}. \tag{21}$$

Note that the function $\rho(x,x')$ in (17) is the reflection function from a semi-infinite isothermic medium. Since we have presented this later in the form of eq. (9), we can write, instead of eq. (17),

$$\frac{2}{\lambda}\alpha(x')P(0,x',x)=\sum_{k=0}^{\infty} A_k\alpha_k(x')\varphi_k(x)+\sum_{m=0}^{\infty} B_m\omega_m(x')\psi_m(x). \tag{22}$$

Let us multiply eq. (20) by $\alpha_k(x)$ and integrate with respect to x over the range $(-\infty\ \infty)$. Repeating the same operation for $\omega_m(x)$ yields, for $R_n^k=\int\alpha(x')R_n(x')dx$, the following system of the algebraic equations:

$$R_n^k=\sum_{k=0}^{\infty} A_k R_n^k\xi_k+\sum_{m=0}^{\infty} B_m R_n^m\sigma_{k,m}+\delta_k^{(n-1)} \tag{23}$$

$$R_n^m=\sum_{k=0}^{\infty} A_k R_n^k\mu_{m,k}+\sum_{m=0}^{\infty} B_m R_n^m\theta_m+\zeta_m^{(n-1)}, \tag{24}$$

where we have introduced the notation

$$\xi_k = \int_{-\infty}^{\infty} \frac{\alpha_k(x)\varphi_k(x)}{v(x)}dx,\ \sigma_{k,m} = \int_{-\infty}^{\infty} \frac{\alpha_k(x)\psi_m(x)}{v(x)}dx,$$
$$\delta_k^{(n)} = \beta \int_{-\infty}^{\infty} \frac{\alpha_k(x)}{v(x)}R_n(x)dx,\ \mu_{m,k} = \int_{-\infty}^{\infty} \frac{\omega_m(x)\varphi_k(x)}{v(x)}dx,$$
$$\theta_m = \int_{-\infty}^{\infty} \frac{\omega_m(x)\psi_m(x)}{v(x)}dx,\ \zeta_m^{(n)} = \beta \int_{-\infty}^{\infty} \frac{\omega_m(x)}{v(x)}R_n(x)dx.$$

Apparently the question is reduced to solving the system of algebraic equations (23–24). It is obvious that we can add different absorption/scattering centers (atoms, molecules, etc.) to the medium, and if their redistribution functions allow bilinear expansions, the problem can be reduced to a system of algebraic equations. For a medium containing two kinds of absorbing/scattering centers, one needs to construct and tabulate functions $\varphi_k(x)$ and $\psi_m(x)$. Once this has been done, one needs to compute the constants $\xi_k, \sigma_{k,m}, \delta_k^{(n)}, \mu_{m,k}, \theta_m$, and $\zeta_m^{(n)}$, after which the line profiles are evaluated from eqs. (20) and (21) using recursion relations. This discussion clearly remains valid for more than two kinds of absorbing/scattering centers in the medium (provided that their redistribution functions allow a bilinear expansion) and for more general assumptions relative to the geometry of the medium.

4 Discussion

It is known that coherent scattering in a medium gives the source function a non-LTE character. This fact alone makes it possible for spectral lines to appear in emission, even if the *line* source function does not deviate from LTE[1]. The mechanism requires a low temperature gradient, a strong spectral line and an LTE process of line formation. The criteria derived for the appearance of the lines in emission in a non-isothermal stellar atmosphere are much more stringent and it is possible that this mechanism cannot work in a *real* stellar atmosphere. Another difficulty in producing emission lines in a *normal* atmosphere is the small cross-section of the Thomson scattering. However, in the general case, Thomson scattering on free electrons is not necessarily the only nor the most effective mechanism among the scattering processes which may exist in astrophysical plasmas.

We have computed line profiles from a semi-infinite, one-dimensional atmosphere assuming complete redistribution on atoms and coherent scattering on

free electrons. For the isothermal and non-isothermal (n=1) cases we have numerically solved the systems of equations (10–12) and (23–24), respectively. Problems connected with the method of approximation for the solution of the system of equations of the form (10–12) are discussed in detail in the work of Haruthyunian and Nikoghossian[6].

Figures 1 and 2 illustrate the absorption/emission profiles for different values of γ, λ, and β. The profiles for the isothermal and non-isothermal (n=1) media are shown in Figs. 1 and 2, respectively. To compute the B_n coefficients in the sum (15) we have considered a hypothetical line at 6000 Å, a stellar atmosphere with $T_{\rm eff} = 40000$ K and a gray-atmosphere temperature law. We can see that even in the case of a non-LTE source function in the line (i.e., $\lambda < 1$) one can have emission lines depending on the efficiency of scattering and absorption in continuum. The five parameters involved in this problem are the scattering (γ) and the absorption (β) in continuum, the scattering (λ) and the absorption ($\alpha(x)$) in the line and the temperature gradient in the medium. The addition of scattering to the atmosphere can raise or lower the local continuum depending on the location of the spectral line, the efficiency of scattering and a temperature gradient[10]. Therefore, the variations of continuum in Figs. 1 and 2 have only a *local* character. We have already seen[10] that the variation of the continuum due to the scattering is nothing else than a *redistribution* of emergent radiation demanded by the condition of radiative equilibrium and the conservation of integral flux. Our Fig. 2 clearly demonstrates that the conditions for emission are much more stringent in the non-isothermal than in the isothermal atmosphere. It is also possible to have a situation in which the wings (formed in LTE deep in the atmosphere) of a strong line will appear in emission while the line core is in absorption because of the strong non-LTE effects. Interesting effects may appear when one studies a wing and a core of the spectral line separately, considering depth and frequency dependence of γ, β, and λ.

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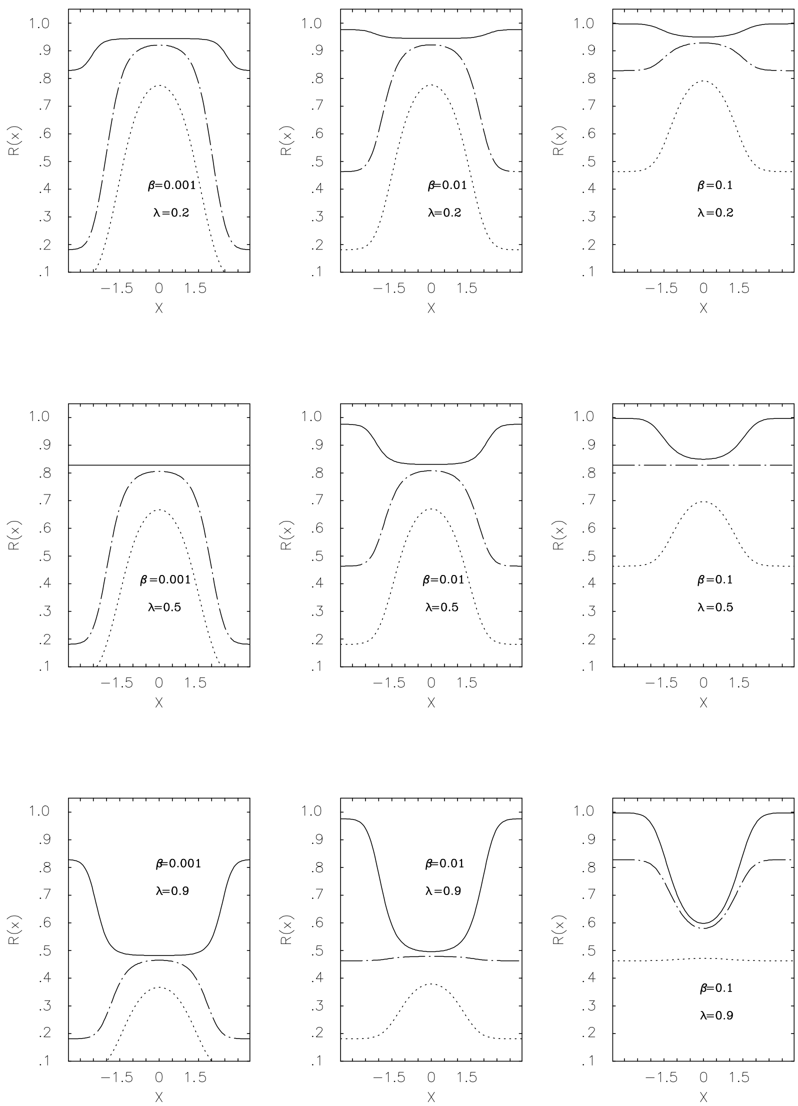

Fig. 1. Spectral line profiles in the case of complete frequency redistribution on atoms and coherent scattering in continuum for an isothermal atmosphere. Computations have been done for γ=0.001 (solid line), γ=0.1 (dash-dotted line) and γ=1.0 (dotted line). Values of β and λ are indicated.

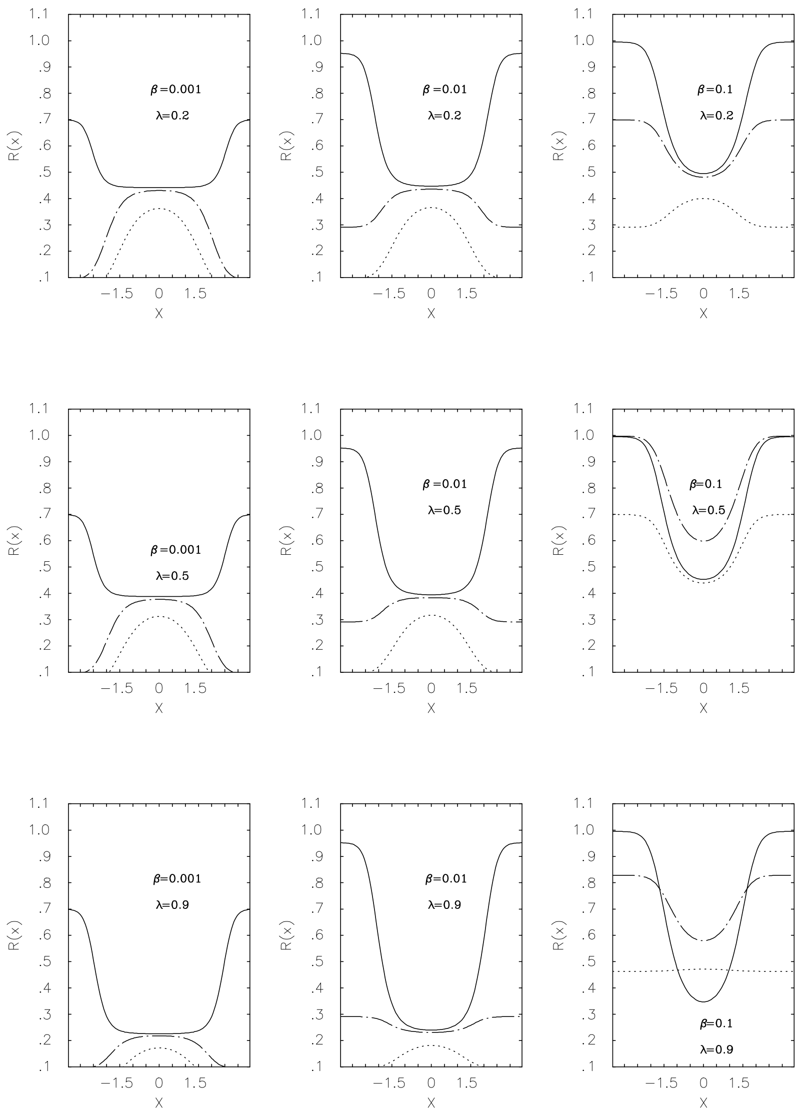

Fig. 2. The same as Fig 1. but for a non-isothermal atmosphere with $n = 1$ (see text).